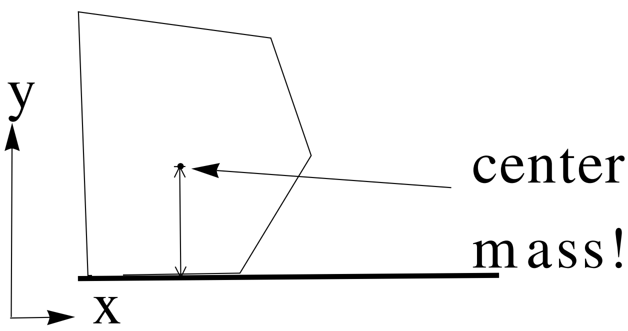
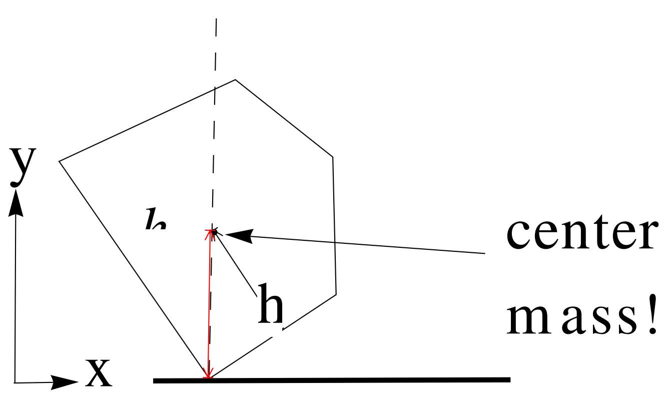
Let's consider the physics of tipping a box . . . I'm using *Mathematica* to type this up and solve equations along the way.



I'm drawing a box badly on purpose - the exact shape of the box does not matter in the calculations I'm doing

Convince yourself that there is such thing a center of mass (there is!)

When does the box tip? Well, it can tip if I can draw straightline through the center of mass and single point on the edge of the box that touchs the ground, if you don't believe me try drawing it. Interesting side note - a ball is always ready to tip over if we consider it as a box with ∞ many sides, except we call a ball tipping forward "rolling".



I realize that I could do this problem in several ways, I'm choosing to simply look at energy conservation. I will assume that a skater transfers ALL of his kinetic energy into the box and that the box arrives at this critical tipping point. This will allow us to find the maximum base of the box that <u>will</u> be tipped as described, so any box with a wider base but otherwise the same <u>will not</u> tip over, and a box with a smaller base will.

The initial potential energy of the box and skater

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U0 = M g h + g m Y;
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The initial kinetic energy of the skater

$$K = \frac{1}{2} m v^2;$$

The final energy of the box + skater is, notice that skater is balanced on top of this about-to-tip-over-box, I have already assumed a rectangular box I guess

$$Uf = M g\, h_{new} + g\, m\, 2\, h_{new};$$

Energy is conserved so:

$$Eqn = U0 + K == Uf;$$

$$Eqn$$

$$\frac{m v^2}{2} + g\, m\, Y + \frac{g M Y}{2} == 2\, g\, m \sqrt{\frac{X^2}{4} + \frac{Y^2}{4}} + g\, M \sqrt{\frac{X^2}{4} + \frac{Y^2}{4}}$$

At this point we need to relate the h_{new} and h to the dimensions of the box, let just take a simple rectangular box .. to lazy to draw this right now, let let the base be Xand the total height Y. I'm assuming a box of uniform density.

$$h = \frac{Y}{2};$$

$$h_{new} = \sqrt{h^2 + \left(\frac{X}{2}\right)^2};$$

$$Eqn$$

$$\frac{m v^2}{2} + g\, m\, Y + \frac{g M Y}{2} == 2\, g\, m \sqrt{\frac{X^2}{4} + \frac{Y^2}{4}} + g\, M \sqrt{\frac{X^2}{4} + \frac{Y^2}{4}}$$

$$Eqn\ /.\ \{Y \to 0\}$$

$$\frac{m v^2}{2} == g\, m \sqrt{X^2} + \frac{1}{2} g\, M \sqrt{X^2}$$

we can solve for X- the base

$$Solve[Eqn,\ X]$$

$$\left\{\left\{X \to -\left(\sqrt{m}\ v \sqrt{\left(m v^2 + 4 g m Y + 2 g M Y\right)}\right) \Big/ \left(g \sqrt{4 m^2 + 4 m M + M^2}\right)\right\},\right.$$
$$\left.\left\{X \to \left(\sqrt{m}\ v \sqrt{\left(m v^2 + 4 g m Y + 2 g M Y\right)}\right) \Big/ \left(g \sqrt{4 m^2 + 4 m M + M^2}\right)\right\}\right\}$$

we take the positively values solution

$$\frac{\sqrt{m}\ v \sqrt{m v^2 + 4 g m Y + 2 g M Y}}{g \sqrt{4 m^2 + 4 m M + M^2}}$$

let simplify and plot it

$$\texttt{Simplify}\left[\left(\sqrt{m}\ v\sqrt{\left(m v^2+4 g m Y+2 g M Y\right)}\right)\Big/\left(g\sqrt{4 m^2+4 m M+M^2}\right),\ \{m>0,\ M>0\}\right]$$

$$\left(v\sqrt{\left(m\left(2 g M Y+m\left(v^2+4 g Y\right)\right)\right)}\right)\Big/\left(g\left(2 m+M\right)\right)$$

let's set $M = \mu\, m$

$$\texttt{Apart}\left[\left(v\sqrt{\left(m\left(2 g M Y+m\left(v^2+4 g Y\right)\right)\right)}\right)\Big/\left(g\left(2 m+M\right)\right)\ /.\ \{m\to \mu\, M\}\right]$$

$$\left(v\sqrt{\left(M^2\mu\left(2 g Y+v^2\mu+4 g Y\mu\right)\right)}\right)\Big/\left(g M\left(1+2\mu\right)\right)$$

$$\texttt{Apart}\left[\%^2\right]$$

$$\frac{v^2\left(v^2+4 g Y\right)}{4 g^2}+\frac{v^4}{4 g^2\left(1+2\mu\right)^2}+\frac{-v^4-2 g v^2 Y}{2 g^2\left(1+2\mu\right)}$$

μ is just the ratio of skaters mass to the box mass, definetly smaller that 1 ! I think its roughly .3 for Frank's box, we can also see that $\frac{v^2}{g}$ is a natural variable for this equation. Physicists are trained to reduce the number of free variables whenever possible.

$$\texttt{Simplify}\left[\left(\frac{v^2\left(v^2+4 g Y\right)}{4 g^2}+\frac{v^4}{4 g^2\left(1+2\mu\right)^2}+\frac{-v^4-2 g v^2 Y}{2 g^2\left(1+2\mu\right)}\right)\ /.\ \left\{g\to \chi\, v^2\right\}\right]$$

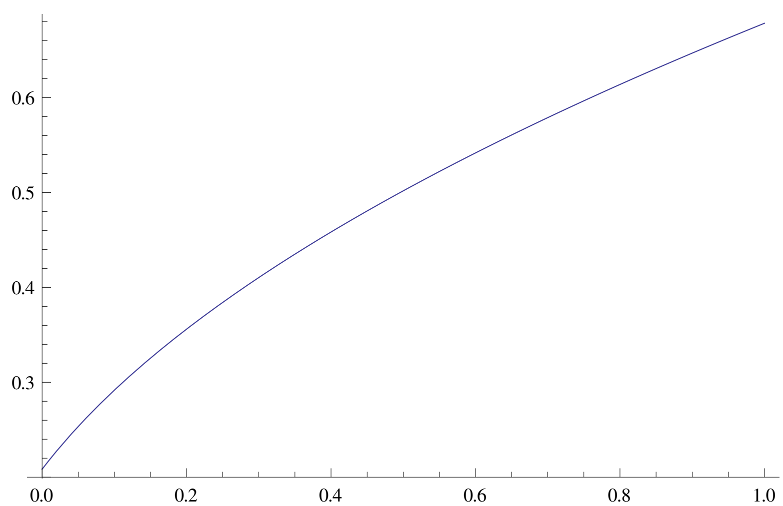
$$\frac{\mu\left(\mu+2 Y\chi+4 Y\mu\chi\right)}{\left(\chi+2\mu\chi\right)^2}$$

$$\texttt{Base} = \sqrt{\frac{\mu\left(\mu+2 Y\chi+4 Y\mu\chi\right)}{\left(\chi+2\mu\chi\right)^2}}\ ;$$

What is a reasoable value fo χ, convince yourself that its $\approx \frac{10}{3} = .9$

Plot the result in cm, min safe base size on the y-axis

$$\texttt{Plot}[\texttt{Base}\ /.\ \{\chi\to .9,\ \mu\to .3\},\ \{Y,\ 0,\ 1\}]$$



It looks like I over estimated the base width earlier by a factor of ~2, if your base now is wider that this now you obviously don't want to but it on top of a smaller platform, that would change the problem. I also haven't compensated for the offcenter center of mass . . . if you remind me I'll do that later.

Let's add some impact absorbption to the skaters knees - say the skater can comfortably jump of off some box that is αY tall. if the skater can (I hope) jump of the grindbox we are using without getting hurt set $\alpha \approx 1$

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UAbsorb = m g Y;
Eqn = U0 + K == Uf + UAbsorb
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$$\frac{m v^2}{2} + g m Y + \frac{g M Y}{2} == g m Y + 2 g m \sqrt{\frac{X^2}{4} + \frac{Y^2}{4}} + g M \sqrt{\frac{X^2}{4} + \frac{Y^2}{4}}$$

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Solve[Eqn, X]
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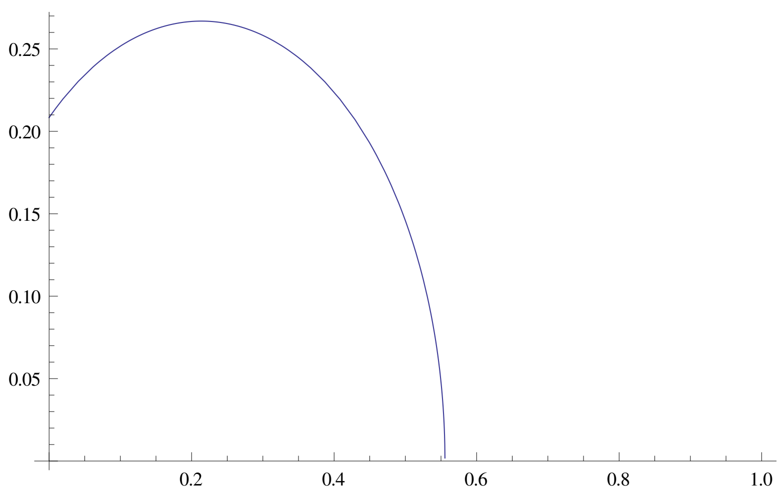
$$\left\{\left\{X \to -\left(\sqrt{m}\sqrt{v^2 - 2 g Y}\sqrt{(m v^2 + 2 g m Y + 2 g M Y)}\right) \Big/ \left(g\sqrt{4 m^2 + 4 m M + M^2}\right)\right\}, \left\{X \to \left(\sqrt{m}\sqrt{v^2 - 2 g Y}\sqrt{(m v^2 + 2 g m Y + 2 g M Y)}\right) \Big/ \left(g\sqrt{4 m^2 + 4 m M + M^2}\right)\right\}\right\}$$

some computer aided algebra that I don't want to do on paper. . .

$$\sqrt{\text{Simplify}\left[\text{Apart}\left[\left(\left(\sqrt{m}\sqrt{v^2 - 2 g Y}\sqrt{(m v^2 + 2 g m Y + 2 g M Y)}\right) \Big/ \left(g\sqrt{(4 m^2 + 4 m M + M^2)}\right)\right)^2\right] /. \{m \to \mu M, g \to \chi v^2\}\right]}$$

$$\sqrt{\left(-(\mu\ (-1 + 2 Y \chi)\ (\mu + 2 Y \chi + 2 Y \mu \chi)) \Big/ (\chi + 2 \mu \chi)^2\right)}$$

$$\text{Plot}\left[\sqrt{\left(-(\mu\ (-1 + 2 Y \chi)\ (\mu + 2 Y \chi + 2 Y \mu \chi)) \Big/ (\chi + 2 \mu \chi)^2\right)} /. \{\chi \to .9, \mu \to .3\}, \{Y, 0, 1\}\right]$$



Interesting, if the skater absorbs enough energy in his knees he can make do with a vanishingly small base . . good luck

This is close to how far you can get with simple energy analysis, we can consider a more dynamical picture where the details of how a skater transfers momentum matter.